Research notes
Information Theory
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Zhang Berger region is the same as Paul Cuff region

Note: This result is mentioned by Paul Cuff in his paper on co-ordination capacity.

Consider the Broadcast co-ordination problem shown in Fig. 1 X^n $p_0(x)$ is a source given by nature and is available at the encoder and a distribution $p(y,z|x)$ is desired. The encoder sends messages $M_1 \in [1 : 2^{nR_1}]$ and $M_2 \in [1 : 2^{nR_2}$ to decoders 1 and 2 respectively. Decoder 1 produces an output $Y^n(M_1)$ and decoder 2 produces an output $Z^n(M_2)$. The purpose is to determine the necessary and sufficient set of rate pairs (R_1, R_2) such that $|P_{X^n,Y^n,Z^n}(x,y,z) - p(x,y,z)|_{TV} \to 0$ where $p(x,y,z) = p_0(x)p(y,z|x)$.

The Paul Cuff achievable region is given by

$$\begin{aligned}
R_1 &\geq I(X;U) + I(X;Y|U) = I(X;YU) \\
R_2 &\geq I(X;U) + I(X;Z|U) = I(X;ZU) \\
R_1 + R_2 &\geq I(X;Y|U) + I(X;Z|U) + 2I(X;U) + I(Y;Z|XU) \\
&= I(X;YU) + I(X;ZU) + I(Y;Z|XU)
\end{aligned}$$

where $p(u,x,y,z) = p_0(x)p(u,y,z|x)$.

The Zhang-Berger region is given by

$$\begin{aligned}
R_1 &\geq I(X;U) + I(X;Y|U) = I(X;YU) \\
R_2 &\geq I(X;U) + I(X;Z|U) = I(X;ZU) \\
R_1 + R_2 &\geq I(X;Y|U) + I(X;Z|U) + 2I(X;U) + I(Y;Z|XU)
\end{aligned}$$

where $p(u,x,y,z) = p_0(x)p(u,y,z|x)$.

Note that using chain rule for mutual information successively, we have

$$\begin{aligned}
&I(X;Y|U) + I(X;Z|U) + 2I(X;U) + I(Y;Z|XU) \\
&= I(X;Y|U) + 2I(X;U) + (I(X;Z|U) + I(Y;Z|XU)) \\
&= I(X;Y|U) + 2I(X;U) + I(XY;Z|U) \\
&= I(X;Y|U) + I(XY;Z|U) + 2I(X;U) \\
&= I(X;Y|U) + I(X;Z|U) + I(Y;Z|XU) + 2I(X;U)
\end{aligned}$$

This shows that Paul Cuff achievable region is the same as the Zhang-Berger region. However the achievability schemes seem to differ considerably.

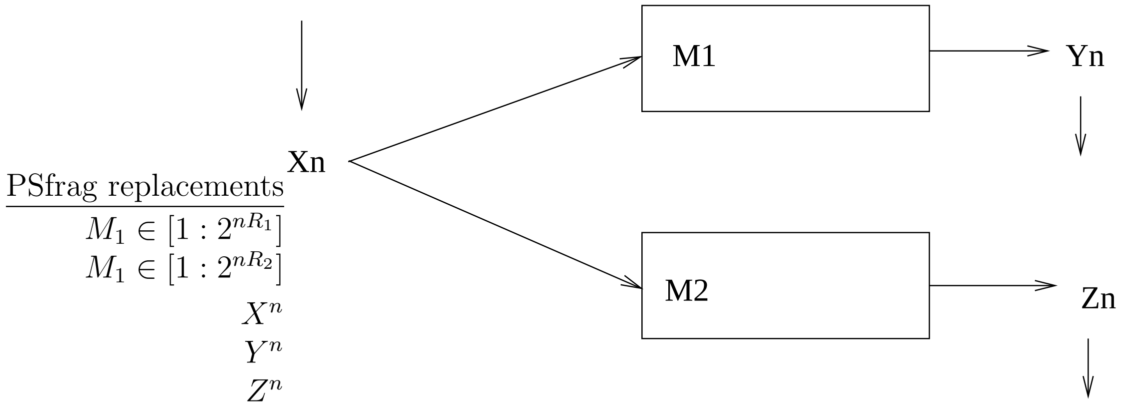

Figure 1: Broadcast co-ordination (c.f. Multiple Descriptions problem)