Interference Alignment and Spatial Degrees of Freedom for the K User Interference Channel

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Can interference effects be mitigated?

Regardless of how many speakers and listerners are located within earshot of each other, can each speaker speak half the time and be heard without any interference by its intended receiver?

Can interference effects be mitigated?

Answer: YES.

- Assume propagation delay T_{ij} is different for each transmitter-receiver pair i, j.
- Assume T_{ij} is odd when message from transmitter i is desired by receiver j and is even otherwise.
- Each transmitter speaks during odd time slots and stays silent at other times.
- All interference is aligned in even time slots.

Channel Model

The channel output at the kth receiver over the fth frequency slot and tth time slot is given by

$$Y^{[k]}(f,t) = H^{[k1]}(f)X^{[1]}(f,t) + \cdots + H^{[kK]}(f)X^{[K]}(f,t) + Z^{[k]}(f,t)$$

where, $k \in \{1, 2, \ldots, K\}$ is the user index, $f \in N$ is the frequency slot index, $t \in N$ is the time slot index, $Y^{[k]}(f,t)$ is the output signal of the kth receiver, $X^{[k]}(f,t)$ is the input signal of the kth transmitter, $H^{[kj]}(f)$ is the channel fade coefficient from transmitter j to receiver k over the fth frequency slot and $Z^{[k]}(f)$ is the $N(0,1)$ AWGN term at the kth receiver.

Degrees of freedom

Capacity region $C(\rho)$ is defined as the set of achievable rate-vectors $R_1, R_2, \ldots R_K$ for the K users with a total transmit power constraint ρ. Degree of freedom region $\mathcal{D}$ is as follows:

$$\mathcal{D} = \Big\{ (d_1, \ldots, d_K) \in R_+^K : \forall (w_1, \ldots, w_K) \in R_+^K$$

$$w_1 d_1 + w_2 d_2 + \ldots w_K d_K \leq$$

$$\limsup_{\rho \to \infty} \left[\sup_{R(\rho) \in C(\rho)} \frac{[w_1 R_1(\rho) + \ldots w_K R_K(\rho)]}{log(\rho)} \right] \Big\}$$

Theorem

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The number of degrees of freedom for the K user interference channel with single antennas at all nodes is $K/2$.

Achievability for 3 users

Consider $2n+1$ frequency slots with independent channel coefficients. $\bar{X}^{[k]}(t)$, $\bar{Y}^{[k]}(t)$, $\bar{Z}^{[k]}(t)$ are $2n+1$ extensions, where $k \in \{1,2,3\}$ denotes the user index. $H^{[kj]}$ is a $2n+1 \times 2n+1$ diagonal channel matrix for $k, j \in \{1,2,3\}$. In the extended channel, message W_1 at transmitter 1 is encoded in to $n+1$ independent streams $x_m^{[1]}(t), m = 1, \ldots 2n+1$ so that

$$\bar{X}^{[1]}(t) = \sum_{m=1}^{n+1} x_m^{[1]}(t) v_m^{[1]} = \bar{V}^{[1]} X^{[1]}(t)$$

Similarly W_2 and W_3 are coded in to n streams.

Achievability

The received signal at receiver i is given by:

$$\bar{Y}^{[i]}(t) = \bar{H}^{[i1]}\bar{V}^{[1]}\bar{X}^{[1]}(t) + \bar{H}^{[i2]}\bar{V}^{[2]}\bar{X}^{[2]}(t) + \bar{H}^{[i3]}\bar{V}^{[3]}\bar{X}^{[3]}(t)$$

For interference alignment we require to choose V's such that:

$$rank([\bar{H}^{[13]}\bar{V}^{[3]} \quad \bar{H}^{[12]}\bar{V}^{[2]}]) \leq n$$
$$rank([\bar{H}^{[23]}\bar{V}^{[3]} \quad \bar{H}^{[21]}\bar{V}^{[1]}]) \leq n+1$$
$$rank([\bar{H}^{[31]}\bar{V}^{[1]} \quad \bar{H}^{[32]}\bar{V}^{[2]}]) \leq n+1$$

and that $[\bar{H}^{[i1]}\bar{V}^{[1]}\bar{H}^{[i2]}\bar{V}^{[2]}\bar{H}^{[i3]}\bar{V}^{[3]}]$ is of full rank. This can be proved possible using linear algebra for an appropriate choice of V's.

Converse

A. Host-Madsen and A. Nosratinia, “he multiplexing gain of wireless networks,” in Proc. of ISIT, 2005.

MIMO-X channel

- 2 transmitter-receiver pairs.
- M antennas per each transmitter and receiver.
- Sum-rate degrees of freedom d satisfies $\lfloor \frac{4}{3}M \rfloor \leq d \leq \frac{4}{3}M$.

S. A. Jafar and S. Shamai, "Degrees of Freedom Region of the MIMO X. Channel," IEEE TRANSACTIONS ON INFORMATION THEORY, VOL. 54, NO. 1, JANUARY 2008, Page(s): 151-170.