EE360 project: Interference Alignment for MIMO interference channel using Imperfect CSI

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Abstract

Interference alignment is a novel technique introduced in [1] that allows for K/2 degrees of freedom in a K-user interference channel. A related work [2] proves the achievability of 4/3 degrees of freedom on the 2 user MIMO-X channel. However, the techniques require Channel State Information (CSI) at transmitter. In [3], it is shown that $M(L-1)\log P$ bits of feedback broadcased by each Rx to all Tx suffices to achieve the stated K/2 degrees of freedom for the K user interference channel. In this report I show that $\frac{1}{2}\log P + \textit{constant}$ bits of feedback per channel coefficient suffice to achieve dof for a K user MIMO interference channel. The proof uses the achievability presented in [5]. Finally I discuss the limitations of the proposed scheme and suggest possible future directions.

1 Prior work

Cadambe and Jafar [1] propose a novel method in which K transmitter-receiver pairs possibly interfering with each other can communicate for 30 minutes each in one hour without effects of interference. They achieve this by alignment of interference in one dimension and placement of the signal in another orthogonal dimension. The disadvantage of the proposed technique is that it assumes perfect CSI at both transmitters and receivers. The capacity can be achieved at high SNRs using only simple linear codes. A related work by Jafar and Shamai [2] considers the case of 2 transmitter-receiver pairs with M antennas for each transmitter and receiver. They prove that $\lfloor \frac{4}{3}M \rfloor$ degrees of freedom can be achieved. They also used linear codes to achieve the proposed $\lfloor \frac{4}{3}M \rfloor$ degrees of freedom.

Recently Thukral and Bolcskei [3] presented a technique that assumes perfect CSI at receivers, but imperfect CSI at transmitters and still implements interference alignment successfully. They proved that $K/2$ degrees of freedom can be achieved for the K-user interference channel provided the number of feedback bits broadcast by each destination is atleast $M(L-1)\log P$. (The authors assume frequency selective channels with L taps each). In another work, Ghasemi, Motahari and Khandani [5] investigate the achievable degrees of freedom for the more general K user MIMO interference channel with M transmit antennas per transmitter and N receive antennas per receiver. They obtain inner and outer bounds and special cases when the bounds coincide. They assume perfect CSI at both transmitter and receiver.

In [4], the authors present a new achievability scheme for interference alignment. They exploit the fact that a real line consists of infinite rational dimensions. They do away with the requirement of independant channels for interference alignment. In [5], the authors use the proposed technique to achieve the optimal dof of MIMO interference channels for large number of users.

This report is organized as follows: In section 2, we discuss intuitively why limited feedback should suffice. In section 3, we briefly outline the technique presented in [4]. In section 4, we show how the technique can be used to achieve interference alignment with negligible feedback. In the last section we discuss limitations of the proposed scheme and possible future directions for research.

2 How much Feedback is necessary?

A brief intuitive analysis of the results in [3] is as follows: Suppose that one wants to evaluate a function $f(x, y)$. If x is known only to n bit accuracy, one can intuitively argue that that knowing y to more than n bit accuracy will in general not be that benificial in improving the accuracy of $f(x, y)$. Following this line of reasoning, and the fact that it makes information theoretic sense to "know" the signal only up to an accuracy of $\log P$ bits, it makes sense that each of the channel coefficients $h_{ij}(m)$ need be known by the transmitter to an accuracy of only $\log P$ bits. It follows immediately that each receiver needs to broadcast only $ML \log P$ bits (Since there are M transmit antennas and L taps each). A careful analysis of the quantization error formally proves the result.

3 A new technique for Interference Alignment

In this section, we present a brief overview of the technique presented in [4] that can be used to achieve the dof for Gaussian Interference Channels, uplink channels in cellular systems and the MIMO-X channel.
The technique presented here differs from the above techniques in that instead of considering an n-dimensional Euclidean space (n independent channels), one only needs to realize that a single real line has infinite rational dimensions.
First we state a theorem.

Theorem 1 *Let $a_1, a_2, \dots a_n$ be rationally independent real numbers, i.e there is no non-trivial integer solution of the equation $x_0 + \sum_1^n a_i x_i = 0$ with $x_i \in \mathcal{Z}$. Suppose that we have a set*

$$\mathcal{A} = \left\{ x_0 + \sum_{i=1}^{n} a_i x_i | -Q \le x_i \le Q, x_i \in \mathcal{Z} \right\}$$

Then the minimum absolute difference between 2 elements of the above set is of the order c/Q^n where c is some constant. This theorem, termed Khintchine-Groshev theorem, is valid for all ordered tuples $(a_1, a_2, \dots a_n)$ except for a set of measure 0.

Now consider a MIMO-X channel with 2 senders and 2 receivers. The channel is defined by the following equations:

$$y_1 = h_{11}x_1 + h_{12}x_2 + z_1$$
$$y_2 = h_{21}x_1 + h_{22}x_2 + z_2$$

One can choose the signalling scheme as follows:

$$x_1 = h_{22}u_1 + h_{12}v_1 \quad (1)$$
$$x_2 = h_{21}u_2 + h_{11}v_2 \quad (2)$$

where u_1, u_2, v_1, v_2 are integer valued signals ranging from $-Q$ to Q. As usual h_{ij} is the channel coefficient from transmitter j to receiver i. u_1, u_2 are intended by receiver 1 whereas v_1, v_2 are intended by receiver 2.
In that case one obtains:

$$y_1 = (h_{11}h_{22})u_1 + (h_{21}h_{12})u_2 + (h_{11}h_{12})(v_1 + v_2) + z_1 \quad (3)$$

$$y_2 = (h_{21}h_{12})v_1 + (h_{11}h_{22})v_2 + (h_{21}h_{22})(u_1 + u_2) + z_2 \quad (4)$$

The way interference alignment works is highlighted by the above equation. Each real number acts as a dimension, and each integer coefficient multiplying it acts as the co-ordinates of the signal along that dimension. In the presence of the noise term, one needs a minimum distance between two points (each point corresponding to an integer triplet) for successfully decoding the integers u_1, u_2, v_1, v_2. This imposes the additional constraint that there are 1/3 dof per stream. Thus with 4 usable streams, a dof of 4/3 is achievable.
To see why each stream has 1/3 dof, we apply Khintchine-Groshev theorem as follows: The signal power is roughly Q^2 where Q is the largest integer. The allowed noise power is $(1/Q^2)^2$, the square of the minimum distance between 2 signal points in the constellation. Hence, $0.5\log(SNR) = 3\log Q$. The achievable rate R with $2Q$ possible integers is of the order $\log Q$. Hence for large Q, the dof of the stream is $\frac{R}{0.5\log(SNR)} = \frac{1}{3}$. The authors give a more formal description of the theorems [4]. A remarkable fact to be observed here is that the signal constellation fits in a 1-d Euclidian space, but this space has 3 rational dimensions, each with 1/3 dof, at each receiver.
The above technique can be extended for the K user MIMO Interference Channel as shown in [5].

4 Feedback requirement for the general case

In this section, we show how one can achieve Interference Alignment with $\frac{1}{2}\log P$ bits of feedback per channel coefficient using the above technique.
Note in equn. (3) that the channel has been split in to 3 rational dimensions, each of 1/3 dof. In general, the real line is split in to $n+1$ rational dimensions, each of dimension $1/n+1$. Each dimension is a monomial in the channel coefficients. By Khintchine-Groshev theorem, the noise is $O(1/Q^n)$. Let δ be the maximum relative error in h_{ij}'s. A monomial is of the form $\Pi_{ij}h_{ij}^{s_{ij}}$ where $s_{ij} \in \mathcal{Z}^+$. Hence it has a maximum relative error $\delta \sum s_{ij}$. Since it is premultiplied by an integer signal of maximum value Q, the maximum error caused due to channel uncertainty is $O(Q\delta)$. It doesn't make sense to have this uncertainty smaller than the noise: $O(1/Q^n)$. It follows that one requires $1/\delta = O(Q^{n+1})$. Since feedback requires $\log(1/\delta)$ bits, the required feedback per channel coefficient is $(n+1)\log Q + c$ bits for some constant c. However, as in the previous section, the signal strength is $O(Q^2)$ and the noise power is $O(1/Q^{2n})$. Hence, the SNR is $O(Q^{2n+2})$. Note that $(n+1)\log Q = \frac{1}{2}\log SNR$. Hence the required feedback bits is $\frac{M}{2}\log P + c'$ where M is the number of transmit antennas (and hence the number of channel coefficients measured by each receiver). P is the SNR when the receiver noise is $\mathcal{N}(0,1)$.
The above analysis holds for any class of problems solved by splitting the real line in to rational dimensions including the K user MIMO interference channel presented in [5]. Thus required feedback is $\frac{M}{2}\log P$ bits.

5 Limitations and Future work

On the optimistic side the above technique shows that dof can be achieved for a K user MIMO interference channel with limited feedback.

The proposed scheme has drawbacks however. Khintchine-Groshev theorem only shows that the integer signal at the receiver can be decoded in the presence of noise. It however does not provide an efficient algorithm to achieve the same. Thus the achievability remains an information-theoretic proof and is not yet practical. One also has to take in to account the effect of fading. In fast fading, the channel coefficients change so fast that one may not be able to code for a sufficiently large blocklength before the channel changes. The obvious work-around is multiplexing: waiting for the channel coefficients to repeat and interleaving the bits by multiplexing. However, for large dimensions (large number of channel coefficients), it takes a while before a particular channel condition is repeated, hence the information theoretic achievability may take exponentially long time. If each channel coefficient has 2 states and there are 40 channel coefficients, then there are $2^{40} = 10^{12}$ channel states in total and it takes atleast 3 years before a channel condition repeats! The authors, having assumed a fixed channel state, do not seem to have addressed this issue. The assumptions hold only in case of slow fading.

Also, even though an achievable result is presented for required feedback, the minimum required amount of feedback is not yet obtained and the trade-off between feedback and dof is unknown. In slow fading, which is typically the case in wireless communications, one can conclude that this causes only a negligible overhead in feedback requirements.

A close look at equn. (3) suggests that decoding at the receiver may be treated as a Multi-User-Detection (MUD) problem. The signals u_1, u_2 and $v_1 + v_2$ are combined at Rx 1 with appropriate channel gains. One can consider a scheme in which the u_1 sequence is orthogonal to the u_2, v_1, v_2 sequences. The rest of my project will explore this avenue.

References

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