EE360 project: Interference Alignment for MIMO interference channel using Imperfect CSI

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Abstract

Interference alignment is a novel technique introduced in [1] that allows for K/2 degrees of freedom in a K-user interference channel. A related work [2] proves the achievability of 4/3 degrees of freedom on the 2 user MIMO-X channel. However, the techniques require Channel State Information (CSI) at transmitter. In [3], it is shown that $M(L-1)\log P$ bits of feedback broadcased by each Rx to all Tx suffices to achieve the stated K/2 degrees of freedom for the K user interference channel. In this report I show that $\frac{1}{2}\log P+$constant bits of feedback per channel coefficient suffice to achieve dof for a K user MIMO interference channel. The proof uses the achievability presented in [5]. Then I give a proof outline of the converse result (refer [6]) that $\frac{1}{2}\log P$ bits per channel coefficient is the minimum necessary feedback for no loss in dof. Finally I discuss tradeoff between feedback and dof and possible future research directions.

1 Prior work

Cadambe and Jafar [1] propose a novel method in which K transmitter-receiver pairs possibly interfering with each other can communicate for 30 minutes each in one hour without effects of interference. They achieve this by alignment of interference in one dimension and placement of the signal in another orthogonal dimension. The disadvantage of the proposed technique is that it assumes perfect CSI at both transmitters and receivers. The capacity can be achieved at high SNRs using only simple linear codes. A related work by Jafar and Shamai [2] considers the case of 2 transmitter-receiver pairs with M antennas for each transmitter and receiver. They prove that $\lfloor \frac{4}{3}M \rfloor$ degrees of freedom can be achieved. They also used linear codes to achieve the proposed $\lfloor \frac{4}{3}M \rfloor$ degrees of freedom.

Recently Thukral and Bolcskei [3] presented a technique that assumes perfect CSI at receivers, but imperfect CSI at transmitters and still implements interference alignment successfully. They proved that $K/2$ degrees of freedom can be achieved for the K-user interference channel provided the number of feedback bits broadcast by each destination is atleast $M(L-1)\log P$. (The authors assume frequency selective channels with L taps each). This work was generalized in [6] where the authors also prove the converse result that it is impossible to preserve dof with lesser feedback.

In [4], the authors present a new achievability scheme for interference alignment. They exploit the fact that a real line consists of infinite rational dimensions. They do away with the requirement of independant channels for interference alignment. In [5], the authors use the proposed technique to achieve the optimal dof of MIMO interference channels for large number of users. In [11], the author uses the technique to perform interference alignment at the receiver for a compound broadcast channel with 1 transmitter having M transmit antennas and K single antenna receivers. He shows that only co-operation gain is lost from lack of CSIT; gain from interference alignment can still be obtained.

Converse results for Interference Alignment can be found in [8] and [9]. In the former, the authors consider a two-user interference channel with arbitrary number of antennas at each node and use zero forcing to obtain optimal dof. In the latter, the authors show that the dof for the K-user interference channel is atmost $K/2$. The proof is straightforward once one observes that relays do not help improve the dof of interference channels.
In [7], the authors consider a compound broadcast channel and showed that the difference between the different possible channel coefficients must shrink as $O(P^{-1})$ in order that the system has the same dof as the perfect CSIT case. This is equivalent to arguing that $\log P$ bits of feedback is necessary.
This report is organized as follows: In section 2, we discuss intuitively why limited feedback should suffice. In section 3, we briefly outline the technique presented in [4]. In section 4, we show how the technique can be used to achieve interference alignment with negligible feedback. In section 5, we show converse results for minimum required feedback. In section 6, we discuss dof vs feedback trade-off. In the last section, we conclude and discuss future work.

2 How much Feedback is necessary?

A brief intuitive analysis of the results in [3] is as follows: Suppose that one wants to evaluate a function $f(x, y)$. If x is known only to n bit accuracy, one can intuitively argue that that knowing y to more than n bit accuracy will in general not be that benificial in improving the accuracy of $f(x, y)$. Following this line of reasoning, and the fact that it makes information theoretic sense to "know" the signal only up to an accuracy of $\log P$ bits, it makes sense that each of the channel coefficients $h_{ij}(m)$ need be known by the transmitter to an accuracy of only $\log P$ bits. It follows immediately that each receiver needs to broadcast only $ML \log P$ bits (Since there are M transmit antennas and L taps each). A careful analysis of the quantization error formally proves the result.

3 A new technique for Interference Alignment

In this section, we present a brief overview of the technique presented in [4] that can be used to achieve the dof for Gaussian Interference Channels, uplink channels in cellular systems and the MIMO-X channel.
The technique presented here differs from the above techniques in that instead of considering an n-dimensional Euclidean space (n independent channels), one only needs to realize that a single real line has infinite rational dimensions.
First we state a theorem.

Theorem 1 *Let $a_1, a_2, \ldots a_n$ be rationally independent real numbers, i.e there is no non-trivial integer solution of the equation $x_0 + \sum_1^n a_i x_i = 0$ with $x_i \in \mathcal{Z}$. Suppose that we have a set*

$$\mathcal{A} = \left\{ x_0 + \sum_{i=1}^{n} a_i x_i | -Q \leq x_i \leq Q, x_i \in \mathcal{Z} \right\}$$

Then the minimum absolute difference between 2 elements of the above set is of the order c/Q^n where c is some constant. This theorem, termed Khintchine-Groshev theorem, is valid for all ordered tuples $(a_1, a_2, \ldots a_n)$ except for a set of measure 0.

Now consider a MIMO-X channel with 2 senders and 2 receivers. The channel is defined by the following equations:

$$y_1 = h_{11}x_1 + h_{12}x_2 + z_1$$
$$y_2 = h_{21}x_1 + h_{22}x_2 + z_2$$

One can choose the signalling scheme as follows:

$$x_1 = h_{22}u_1 + h_{12}v_1 \quad (1)$$
$$x_2 = h_{21}u_2 + h_{11}v_2 \quad (2)$$

where u_1, u_2, v_1, v_2 are integer valued signals ranging from $-Q$ to Q. As usual h_{ij} is the channel coefficient from transmitter j to receiver i. u_1, u_2 are intended by receiver 1 whereas v_1, v_2 are intended by receiver 2.

In that case one obtains:

$$y_1 = (h_{11}h_{22})u_1 + (h_{21}h_{12})u_2 + (h_{11}h_{12})(v_1 + v_2) + z_1 \quad (3)$$

$$y_2 = (h_{21}h_{12})v_1 + (h_{11}h_{22})v_2 + (h_{21}h_{22})(u_1 + u_2) + z_2 \quad (4)$$

The way interference alignment works is highlighted by the above equation. Each real number acts as a dimension, and each integer coefficient multiplying it acts as the co-ordinates of the signal along that dimension. In the presence of the noise term, one needs a minimum distance between two points (each point corresponding to an integer triplet) for successfully decoding the integers u_1, u_2, v_1, v_2. This imposes the additional constraint that there are 1/3 dof per stream. Thus with 4 usable streams, a dof of 4/3 is achievable.

To see why each stream has 1/3 dof, we apply Khintchine-Groshev theorem as follows: The signal power is roughly Q^2 where Q is the largest integer. The allowed noise power is $(1/Q^2)^2$, the square of the minimum distance between 2 signal points in the constellation. Hence, $0.5\log(SNR) = 3\log Q$. The achievable rate R with $2Q$ possible integers is of the order $\log Q$. Hence for large Q, the dof of the stream is $\frac{R}{0.5\log(SNR)} = \frac{1}{3}$. The authors give a more formal description of the theorems [4]. A remarkable fact to be observed here is that the signal constellation fits in a 1-d Euclidian space, but this space has 3 rational dimensions, each with 1/3 dof, at each receiver.

The above technique can be extended for the K user MIMO Interference Channel as shown in [5].

4 Feedback requirement for the general case

In this section, we show how one can achieve Interference Alignment with $\frac{1}{2}\log P$ bits of feedback per channel coefficient using the above technique. A similar proof can be found in [6].

Note in equn. (3) that the channel has been split in to 3 rational dimensions, each of 1/3 dof. In general, the real line is split in to $n+1$ rational dimensions, each of dimension $1/n+1$. Each dimension is a monomial in the channel coefficients. By Khintchine-Groshev theorem, the noise is $O(1/Q^n)$. Let δ be the maximum relative error in h_{ij}'s. A monomial is of the form $\Pi_{ij} h_{ij}^{s_{ij}}$ where $s_{ij} \in \mathcal{Z}^+$. Hence it has a maximum relative error $\delta \sum s_{ij}$. Since it is premultiplied by an integer signal of maximum value Q, the maximum error caused due to channel uncertainty is $O(Q\delta)$. It doesn't make sense to have this uncertainty smaller than the noise: $O(1/Q^n)$. It follows that one requires $1/\delta = O(Q^{n+1})$. Since feedback requires $\log(1/\delta)$ bits, the required feedback per channel coefficient is $(n+1)\log Q + c$ bits for some constant c. However, as in the previous section, the signal strength is $O(Q^2)$ and the noise power is $O(1/Q^{2n})$. Hence, the SNR is $O(Q^{2n+2})$. Note that $(n+1)\log Q = \frac{1}{2}\log SNR$. Hence the required feedback bits is $\frac{M}{2}\log P + c'$ where M is the number of transmit antennas (and hence the number of channel coefficients measured by each receiver). P is the SNR when the receiver noise is $\mathcal{N}(0,1)$.

The above analysis holds for any class of problems solved by splitting the real line in to rational dimensions including the K user MIMO interference channel presented in [5]. Thus required feedback is $\frac{M}{2}\log P$ bits.

In [6], the authors use the results of [7] and show that this feedback is pareto-optimal, that is reduction in feed-back causes loss in dof. They also show that if a particular receiver is unable to provide perfect feedback and decides to provide only a fraction f of feedback, the dof of that receiver alone can be reduced by this factor, leaving the other users unaffected.

5 Converse Result

In this section I give a brief intuition as to why $\frac{1}{2}\log P$ bits of feedback per channel coefficient is necessary. Consider a broadcast channel with 1 transmitter having 2 antennas and 2 receivers having 1 antenna each:

$$y = Hx + z$$

where y_1, y_2 represent the received signals of users 1,2 respectively and x_1, x_2 denote the inputs. Suppose there is a power constraint P on each input. The system has dof 2 and to achieve the dof, simple zero forcing suffices: Let $x = H^{-1}\tilde{x}$ where $\tilde{x}$ is the actual signal. Each receiver simply decodes its received signal.

Now suppose that the broadcast channel is compound: i.e. there are two possible realizations of H and the transmitter will now have to code for the worst case realization. In such a case, the dof is reduced as shown in [10] and [11].

Now, what happens if there is a relative error δ in the channel coeffecients availabe to the transmitters? In such a scenario, as long as the determinant $|H|$ is finite and non-zero, the relative error in H^{-1} is upper bounded by $c\delta$ for some constant c. Hence, treating interference as noise, the SNR at the receiver is $\frac{P}{c^2\delta^2 P+1}$ (where $Z \sim \mathcal{N}(0,1)$). In order to retain the degrees of freedom we see that $\delta^2 = O(\frac{1}{P})$ which dictates the required $\frac{1}{2}\log P$ bits of feedback.

If less than $\frac{1}{2}\log P$ bits of feedback are available, it seems plausible that the broadcast channel must be treated as a compound broadcast channel. In that case, the dof has to reduce. A formal proof is found in [7]. A similar converse for interference alignment scenario is found in [6].

6 Dof vs feedback trade-off

In this section, we discuss an achievable trade-off between dof and available feedback. It was shown that $\frac{1}{2}\log P$ bits of CSIT per channel coefficient is necessary for perfect dof. If instead only $\frac{f}{2}\log P$ bits of feedback per channel coefficient is avaiable (for $0 < f < 1$), a simple achievability scheme is to simply use a smaller transmit power P^f instead of power P. This strategy achieves $fK/2$ degrees of freedom for the interference channel. Another strategy is to treat the channel uncertainity as different possible channel states of a compound interference channel and find information theoretic upper bounds on worst case performance. It is not yet known if the approach described is optimal.

A more general problem of interest is when different receivers send different amounts of feedback to each transmitter. The dof computation in this case becomes more intractable. In [6], the authors show that if a particular receiver is unable to provide perfect feedback and decides to provide only a fraction f of feedback, the dof of that receiver alone can be reduced by this factor, leaving the other users unaffected. This is only a step in that direction.

7 Future Work

Real interference alignment scheme presented in [4] is useful in many situations where the original interference alignment technique of [1] does not apply (Refer [11]). However, the scheme has drawbacks. Khintchine-Groshev theorem is a guarantee that decoding is possible, but it does not provide an efficient decoding algorithm. Thus the achievability remains an information-theoretic proof and is not yet practical. A close look at equn. (3) suggests that decoding at the receiver may be treated as a Multi-User-Detection (MUD) problem. The signals u_1, u_2 and $v_1 + v_2$ are combined at Rx 1 with appropriate channel gains. One can consider a scheme in which the u_1 sequence is orthogonal to the u_2, v_1, v_2 sequences. It is not yet known if this direction of research will provide useful insights.

It was shown that $\frac{1}{2}\log P$ bits per channel coefficient is the minimum amount of feedback required for no loss of dof. In slow fading, which is typically the case in wireless communications, one can conclude that feedback is only a negligible overhead. In fast fading scenarios, the results show that feedback rate should grow as fast as the signalling rate in order to preserve dof.
As discussed in the previous section, the tradeoff between dof and feedback is as yet unknown. Some attempts in that direction were made in [6]. It seems plausible that the achievable tradeoff presented in the previous section, when the number of bits per channel coefficient remain uniform, is optimal. Converse results for compound channels will lead to more insights on necessary feedback.

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